

Analysis of the microstructures (“rosettes”) in the superposition of periodic layers

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Abstract. Superpositions of periodic dot screens are largely used in electronic imaging in the field of color printing. In such superpositions the interaction between the superposed layers may cause new structures to appear which did not exist in any of the original layers: macrostructures (also known as moiré patterns) and microstructures (also known as rosettes). While macrostructures are not always generated in the superposition (cf. moiré-free superpositions), microstructures exist practically in any superposition, except for the most trivial cases. In fact, even the macrostructures, whenever they occur, consist of variations in the microstructure of the superposition. In the present paper we investigate the microstructures that appear in the superposition of periodic structures and their properties. We also find the conditions on the superposed layers under which the microstructure of the superposition varies—or remains invariant—when individual layers in the superposition are laterally shifted with respect to each other. © 2002 SPIE and IS&T. [DOI: 10.1117/1.1477442]

1 Introduction

When periodic layers (line gratings, dot screens, etc.) are superposed, new structures of two distinct levels may appear in the superposition, which do not exist in any of the original layers: *macrostructures* and *microstructures*.

The macrostructures, usually known as *moiré patterns*, are much coarser than the detail of the original layers, and they are clearly visible even when observed from a distance. The microstructures, on the contrary, are almost as small as the periods of the original layers (typically, just 2–5 times larger), and, therefore, they are only visible when examining the superposition from a close distance or through a magnifying glass. These tiny structures are also called *rosettes* owing to the various flower-like shapes they often form in the superposition of dot screens (Ref. 1, p. 339).

While macrostructures (moiré effects) have been treated over the years in a large number of references (see, for example, in Refs. 2 and 3), only a few studies have been devoted to the microstructures. However, in spite of their tiny size, the microstructures which occur in the superposition are very rich in detail, and their study appears to be not less fascinating than the study of the macrostructures. As

we can see in Fig. 1, quite attractive rosette forms often appear in the superposition, and a look through a magnifying glass may reveal an amazing, subtle, and delicate microworld, full of surprising geometrical forms.

We will see in this paper that macrostructures and microstructures may coexist in the same superposition. However, while microstructures exist practically in any superposition, except for the most trivial cases, macro moiré effects are not always present (cf. stable and unstable moiré-free states in Sec. 2.3 below). In fact, we will see in Sec. 4.1 that the macrostructures, whenever they exist, are constructed from the microstructures of the superposition.

In the present paper we investigate the microstructures generated in the superposition of periodic layers and their properties both in the image domain and in the spectral domain. Our approach is completely general, and not only limited to the rosette morphology in the classical case used for color printing, the superposition of three screens 30° or 60° apart, which has already been studied in Ref. 1, Ref. 4, pp. 57–59, and Ref. 5. We start in Sec. 2 by establishing the required terminology and mathematical framework for the rest of the paper. We then discuss the behavior of the microstructure in all the different types of superpositions: singular superpositions in Sec. 3, and nonsingular superpositions in Sec. 4. Then, in the remaining sections we proceed to the formal explanation of these phenomena. This also leads us to new, general results concerning the stability of the microstructure under layer shifts in the superposition. We show that shifts of individual layers substantially change the microstructure of the superposition (e.g., from dot-centered rosettes to clear-centered rosettes or vice versa) if and only if the superposition is singular. Several figures and examples taken from the printing world illustrate our discussion throughout the paper.

2 Background and Basic Notions

In this introductory section we briefly review the basic notions and terminology that will be used throughout this paper.

2.1 Properties of the Superposed Layers and Their Fourier Spectra

First of all, let us mention that throughout this work we are only concerned with monochrome, black and white images

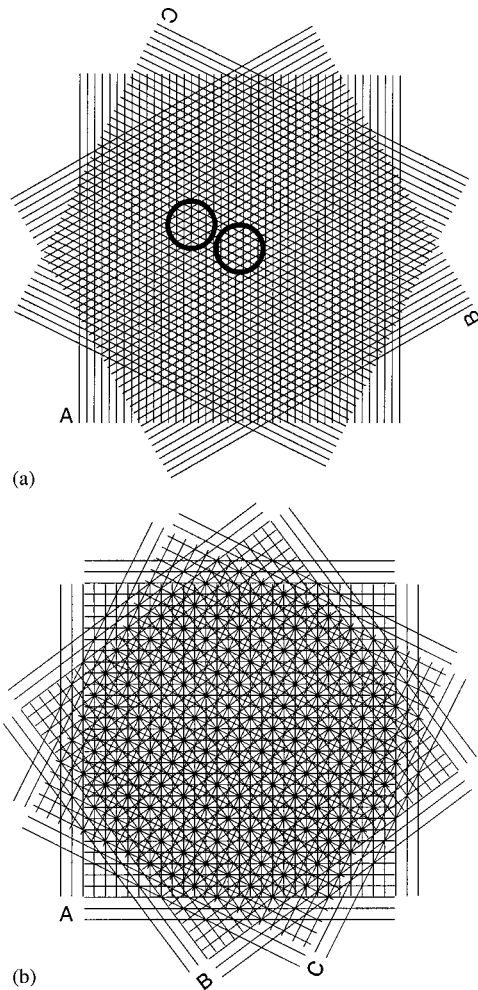


Fig. 1 The superposition of periodic layers may yield very spectacular microstructures (rosettes). (a) A magnification of the three-grating superposition of Fig. 4(h). Note the star-like rosettes which form the bright areas of the macromoiré and the triangular microstructure which forms the darker areas. (b) A magnification of a singular superposition of three grids (=6 gratings) with angles $\theta_1=0^\circ$, $\theta_2=36.8699^\circ$, $\theta_3=63.4349^\circ$, and periods $T_1=T_2$, $T_3=1.118T_1$. This is an example of a periodic, singular superposition (Sec. 3.1).

(or “layers”). This means that each image can be represented by a *reflectance* function, which assigns to any point (x,y) of the image a value between 0 and 1 representing its light reflectance: 0 for black (i.e., no reflected light), 1 for white (i.e., full light reflectance), and intermediate values for in-between shades. In the case of transparencies, the reflectance function is replaced by a *transmittance* function defined in a similar way. The superposition of such images can be done by overprinting, or by laying printed transparencies on top of each other. Since the superposition of black and any other shade always gives black, this suggests a *multiplicative* model for the superposition of monochrome images. Thus, when m monochrome images are superposed, the reflectance of the resulting image is given by the *product* of the reflectance functions of the individual images

$$r(x,y) = r_1(x,y)r_2(x,y)\dots r_m(x,y). \quad (1)$$

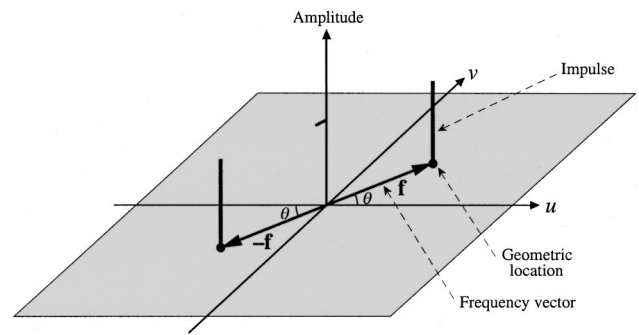


Fig. 2 The *geometric location* and *amplitude* of impulses in the 2D spectrum. To each impulse is attached its *frequency vector*, which points to the geometric location of the impulse in the spectrum plane (u,v) .

According to the convolution theorem (Ref. 6, p. 244) the Fourier transform of the product function is the convolution of the Fourier transforms of the individual functions. Therefore, if we denote the Fourier transform of each function by the respective capital letter and the two-dimensional (2D) convolution by $**$, the spectrum of the superposition is given by

$$R(u,v) = R_1(u,v)**R_2(u,v)**\dots**R_m(u,v). \quad (2)$$

Second, we are basically interested in *periodic* images defined on the continuous (x,y) plane, such as line gratings or dot screens, and their superpositions. This implies that the spectrum of the image on the (u,v) plane is not a continuous one but rather consists of impulses, corresponding to the frequencies which appear in the Fourier series decomposition of the image (Ref. 6, p. 204). In the case of a onefold periodic image, such as a line grating, the spectrum consists of a one-dimensional (1D) “comb” of impulses centered on the origin; in the case of a twofold periodic image the spectrum is a 2D “nail bed” of impulses centered on the origin. Note that we will sometimes use the more general term “cluster” for a comb or a nail bed; this should not be confused, however, with terms such as “clustered dot halftoning,” etc.

Each impulse in the 2D spectrum is characterized by three main properties: its *label* (which is its index in the Fourier series development); its *geometric location* (or *impulse location*), and its *amplitude* (see Fig. 2). To the geometric location of any impulse is attached a *frequency vector* \mathbf{f} in the spectrum plane, which connects the spectrum origin to the geometric location of the impulse. This vector can be expressed either by its polar coordinates (f,θ) , where θ is the direction of the impulse and f is its distance from the origin (i.e., its frequency in that direction); or by its Cartesian coordinates (f_u,f_v) , where f_u and f_v are the horizontal and vertical components of the frequency. In terms of the original image, the *geometric location* of an impulse in the spectrum determines the frequency f and the direction θ of the corresponding periodic component in the image, and the *amplitude* of the impulse represents the intensity of that periodic component in the image. (Note that if the original image is not symmetric about the origin, the amplitude of each impulse in the spectrum may also have a nonzero imaginary component).