CHAPTER 5

VISUAL EQUIVALENCE AND VISUAL MATCHING

5.1 PREAMBLE

Chapters 3 and 4 specify the simple and rigid systems of trichromatic colorimetry and photometry required in practical applications. The pronouncements of the systems are exact only for one or another of the hypothetical colorimetric and photometric standard observers. In colorimetry, the CIE 1931 standard colorimetric observer conforms perfectly with the trichromatic generalization in its stronger form including the persistence of full color matches. Its set of three independent color-matching functions is uniquely determined, subject only to an arbitrary nonsingular linear transformation. Similarly in photometry, the CIE 1924 standard photometric observer has a unique luminous efficiency function, which in fact is a particular linear combination of the color-matching functions of the CIE 1931 standard colorimetric observer.

The various factors that make the assumed properties of the CIE colorimetric and photometric standard observers, at best, approximations for actual visual matching are taken into account in the formal CIE scheme only by introducing the following additional standard observers and constraints to the color-matching conditions:

(a) An alternative set of color-matching functions is standardized for use when the matching field exceeds a certain angular size (4° diameter) and when the matching conditions are otherwise such that rod vision may be assumed to be producing only an insignificant distortion of the match that would be obtained in the absence of a rod response. This alternative set of color-matching functions defines the CIE 1964 supplementary colorimetric standard observer,

(b) A second luminous efficiency function is standardized that is not related to either of the two standard sets of color-matching functions, for use in photometry when the scotopic or rod mechanism is judged to be the only, or at least the greatly predominating, factor determining a brightness match. This luminous efficiency function $V'(\lambda)$ defines the CIE 1951 standard observer for scotopic vision.

In this chapter are collected supplementary notes on the methods, results, and laws of visual matching, both for normal and color-defective eyes, which aim at providing a more general picture going beyond the minimal requirements for the setting up of classical colorimetry as embodied in the CIE systems. As the diverse topics covered need to be treated from somewhat different standpoints, the several notes are kept, as far as possible, self-contained.

5.2 CLASSIFICATION OF MATCHING PROCEDURES

In a visual match, a determination is made of two physical stimuli that, in some sense, produce the same visual response. The following descriptive classification of various matching procedures shows the main differences in their objectives and implications; it is not a complete logical scheme of definitions.

5.2.1 Visual Equivalence and Visual Match by Strict Substitution

If, in a visual experiment of observation whose result is determined by the observer's judgment of what is seen, one physical stimulus (test stimulus *A)* can be substituted for another (test stimulus *B)* without affecting the prescribed result, then, with respect to the particular experiment, prescribed result, and observer, the two stimuli are said to be *visually equivalent by strict substitution.* The substitution must be exact; that is, the alternative test stimulus must be imaged on the same retinal area, and for the same duration. All other stimuli, which constitute *the conditioning stimuli,* must be kept the same. They may be applied anywhere in the field, including the possibility that they extend over the retinal area on which the test stimulus is imaged. If they are not constant in time and if the retina is not fully adapted to them when the test stimulus is applied, the time of application of the latter in the time sequence of the observation must be invariable. Furthermore, if the effective ocular entrance pupil for the conditioning stimuli and hence, the corresponding retinal illuminance might be altered as a result of the interchange of test stimuli, this must be obviated by some modification of the experiment (e.g., by the use of an artificial pupil).

All the various factors that determine the visual environment in which the test stimulus is displayed to the observer constitute the display situation of the test stimulus. These factors include the angular size and position of the test area, the duration and epoch in the time sequence of the observation during which the test stimulus is exposed, and, in fact, all factors that must be kept the same when the test stimulus is substituted by another one.

If not already implied by the foregoing, another condition is that the observations with one test stimulus must be entirely distinct from those with the other. This last condition means that the observer's judgment cannot be a direct comparative judgment of the visual effects of the two test stimuli presented in the same experiment, such as: *A* appears brighter than *B*. Instead, it must be an absolute judgment, for example, the naming of an apparent color, or a decision as to the visible presence or absence of an object, not necessarily the test stimulus, in the field. Alternatively, there can be an "internal" comparison such as a judgment of equality in the appearance of the test stimulus and the appearance of a second stimulus forming one of the conditioning stimuli and remaining, therefore, unchanged when test stimulus *B* is substituted for test stimulus *A*.

To define a particular visual equivalence by strict substitution, both the display situation and the observer's visual judgment must be specified. The latter is conveniently regarded as cast in the form of a proposition about some feature of what the observer sees, which can be affirmed or denied. If, for a given test stimulus, the observer affirms the proposition, the test stimulus is accepted as yielding the prescribed result. In practice, repeat observations will normally be made enabling acceptance to be based on a more refined statistical condition such as "affirmation in more than 50% of repeated observations."

Many of the characteristics of the test stimulus, for example, its size, duration, and position in the visual field, are already laid down in the display situation. In most cases, the test stimulus will be produced by the imaging on the retina of a surface in the external field uniformly emitting radiant power. Thus, the principal way in which such a test stimulus can be varied is, in effect, by changing the absolute spectral distribution of the radiant power emitted by the surface. However, in certain cases, the state of polarization of the test stimulus may be varied, or changes may be brought about in the effective ocular entrance pupil (for this stimulus) so as to modify the angular distribution of the stimulus and the retinal illuminance it produces.

In the complete specification of any test stimulus, in any part of the visual field at any time, it would be necessary to know (a) the absolute spectral radiant power distribution of the corresponding external emitting surface; (b) the size and shape of the effective ocular entrance pupil for the stimulus concerned, and its position of entry in the natural pupil of the eye; and (c) the state of polarization of the radiant energy incident on the cornea. Where the effects of changes in the spectral composition are the main interest, as in the present context, the tacit assumption is normally made that the other factors are kept unchanged in any particular investigation, and stimuli are discussed as though they were uniquely defined by the spectral composition of the corresponding external emitting surface. Where these assumptions are not justified [as in Example (e) of the various matching procedures cited in Section 5.2.5], this must be explicitly indicated.

Although the commonest form of test stimulus is the single uniform light patch, less simple forms may be used for which many of the possible propositions about the appearance of a uniform light patch could never be affirmed by the observer. Other propositions, however, that are never true of a uniform patch may be appropriate for various special groups of nonuniform test stimuli. For example, the proposition may assert that "the two similar halves of the test stimulus appear of the same hue although different, possibly, in brightness and saturation." This could be used in establishing a visual equivalence for test stimuli the two halves of which differ in spectral radiant power distributions. Another example would be the assertion that the apparent color difference between the two halves of the test stimulus is equal to that presented in a neighboring retinal area by a similarly divided comparison stimulus forming part of the unchanged conditioning stimuli. Also, a temporal variation in the color appearance of the test stimulus may be embodied in a proposition such as the uniform test patch in the first half of its period of exposure appears of the same brightness although, possibly, of different hue and saturation as compared with its appearance in the second half-period. This again could generate an equivalence relation among suitable test stimuli.

These more complex visual judgments, that are appropriate to types of test stimuli that are not uniform light patches but constant during their period of exposure, are equally admissible with the simpler judgments. When they are applied, the physical specification of the test stimulus becomes correspondingly more complicated and will vary from one application to another. Where necessary, test stimuli that are uniform and constant during their exposure periods will be described as *simple,* whereas the term *complex* will be applied to other test stimuli, namely those with internal structure, spatial or temporal.

The classification of equivalence and matching procedures deployed here has primarily in mind cases where only simple test stimuli are used. But nothing essential is altered when the visual judgment necessarily demands a complex test stimulus. (Two of the examples of matching procedure given in Section 5.2.5 employ complex test stimuli.)

All the test stimuli that, for a given equivalence experiment and observer, yield the prescribed result, constitute an equivalence set in that any member A of the set is visually equivalent by strict substitution to any member B: (A equiv. B). Clearly this equivalence relation is

(i) *reflexive:* (A equiv. A),
(ii) *symmetric:* (A equiv. B) implies (B equiv. A), and
(iii) *Transitive:* (A equiv. B) and (B equiv. C) imply (A equiv. C).

It is also

(iv) *transitive in the wider sense:* if (A equiv. B) and (C equiv. D) then any two of A, B, C, and D are equivalent.

However, whether the members of the equivalence set satisfy any form of linearity law can be decided only empirically for the equivalence experiment and observer in question. A comprehensive linearity property that may hold in a particular case is defined by the following conditions:

(i) if (A equiv. B), then (αA equiv. B), where α is any positive factor, and
(ii) either of the two equivalences (A equiv. B) and ($[A + B]$ equiv. B) implies the other.

This would mean, for example, that if the required result of the equivalence experiment was that the test stimulus had the same hue as a juxtaposed yellow stimulus (one of the conditioning stimuli), then adding any two stimuli that match yellow in hue must again yield a stimulus that matches yellow. This is nearly true. There are, however, certain deviations, notably the Bezold-Briicke effect. On the other hand, if the required result *is* that the *test* stimulus has a different hue from that of the fixed yellow comparison patch, additivity does not hold, because a red and a green test stimulus each differ in hue from the comparison patch, but an appropriate additive mixture of the two will be yellow.

A more restricted form of linearity law holds if the equivalence satisfies the condition that for any three test stimuli, A, B, and $[A + (1- \alpha)B]$, where $0 < \alpha < 1$, the equivalence of any two implies that all three are equivalent. This form is important in the discussion of Maxwellian trichromacy (see Section 5.3.2).

It is important to note that not all procedures defining a visual equivalence by strict substitution would be described as visual matching. Consider, for example, two alternative test stimuli A and B applied at a particular location in the extrafoveal retina. These stimuli may be equivalent by strict substitution in an experiment where the visual judgment concerns the appearance of a fixed light patch imaged on the fovea and forming one of the conditioning stimuli. However, these test stimuli would not normally be said to be in visual match. If the visual judgment relates specifically to the appearance of the test stimulus itself, the resulting equivalence represents a *visual match by strict substitution.* If, for example, the observer has to judge identity of color between each of the two test stimuli *A* and *B* and another stimulus C, where C forms one of the conditioning stimuli, the relation of equivalence between *A* and *B* is also one of visual match.

In the expression *visual match by strict substitution,* a specific meaning is being given to the term visual match. This specific meaning differs from the usual sense of identity, complete or in one or more qualities, in the appearance of two test stimuli presented in the same observation. The context will usually indicate which kind of visual match is intended; but, where necessary, a visual match by strict substitution will be referred to as a *substitution match* or an *indirect match,* as compared with a *direct match.* Thus, in the example at the end of the previous paragraph, the stimuli *A* and *B* are in substitution or indirect match if the stimuli *A* and C, and *B* and *C,* are, respectively, in direct match.

5.2.2 Asymmetric Comparison and Matching; Quasi-Symmetric Matching

A comparison of the visual effects of two test stimuli in which the display situations of the two stimuli are not in all respects the same, is qualified as *asymmetric.* Asymmetric comparison is certainly involved if the alternative test stimuli are not imaged on identical areas of the same retina; if their size, shape, duration, and so forth, differ; if the dispositions in time and space of the conditioning stimuli with respect to the test stimuli are different; or if the observations with the alternative test stimuli are not independent. Thus, the only comparisons of test stimuli not in some respect asymmetric are determinations of equivalence by strict substitution.

A particular kind of asymmetric equivalence is asymmetric matching which is obtained if the following conditions are fulfilled:

(i) The two stimuli are presented to the observer in different display situations in completely independent observations;

(ii) the common proposition to be satisfied by each stimulus is an *absolute judgment* of its appearance as a patch of color, but not excluding complex stimuli, irrespective of overall shape, size, or duration.

For example, the observer may have to assert that the patch has the hue yellow, or that it is perceptibly different from black, or that it is achromatic, or that its two contiguous halves show no color difference, and so on. If the two test stimuli in their respective display situations satisfy the same prescribed proposition, they are deemed to be matched with respect to the appearance quality judged, and represent what may be called an *indirect asymmetric match.* Such matches are clearly closely related to matches by strict substitution. They represent an extension of the latter, because the display situations of the two test stimuli are not the same. They also represent a limitation, because the criterion for match must be an absolute judgment of the appearance of the test stimulus with no explicit reference to the appearance of any other stimulus in the visual field functioning as a comparison stimulus. Any such stimulus, even though it were present in both display situations, would have an appearance subject to possible modification by the other conditioning stimuli in the display, which would not generally be the same in the two cases. However, an internal comparison stimulus might be used if there were good reason for thinking its appearance would be the same in both display situations.

If the proposition embodying the appearance criterion is kept fixed and any number of different display situations are considered, then every combination of test stimulus and its proper display situation that satisfies the prescribed proposition is a member of an equivalence class of (stimulus/display)-combinations. The test stimuli of any two members of such an equivalence class represent a pair in *indirect asymmetric match,* a match that can be represented symbolically by:

$$(A/\text{ds: } x) \;—\; \text{iam: } k \;—\; (B/\text{ds: } y)$$

where

$A \equiv$ stimulus A
$B \equiv$ stimulus B
ds: $x \equiv$ display situation x
ds: $y \equiv$ display situation y
iam: $k \equiv$ indirect asymmetric match with appearance criterion k

Obviously, the relation denoted by the link symbol "iam: k" is reflexive, symmetrical, and transitive, as well as transitive in the wider sense (see Section 5.2.1). But it is not symmetrical to an interchange of test stimuli only. The relation denoted by $(A/\text{ds} : x)$ — iam : k — $(B/\text{ds} : y)$ does not imply $(B/\text{ds} : x)$ — iam : k — $(A/\text{ds} : y)$, nor, in general, does any other law relating to combinations hold that involves separating a test stimulus from its proper display situation.

The equivalence class just defined is made up of subclasses in each of which the display situation is the same and only the test stimulus varies from one combination to another. Each such subclass is the equivalence set of matches by strict substitution for the given display situation and matching criterion. If there is no restriction on the nature or the number of different display situations taken into account, the full asymmetric equivalence class is characteristic of the proposition defining the matching criterion; it can be regarded, in philosophical parlance, as defining *in extension* and in a way that can be communicated to others what the proposition means for the person making the observations.

By contrast with indirect asymmetric matches, direct asymmetric comparisons are derived from an observer's judgments about the appearances of two test stimuli (*A* and *B*) presented in the same observation. This means that the stimulus distribution, in visual field space and in time, during the course of the observation, but with the omission of the test stimuli, will be common to the display situations of both test stimuli. However, the positions of the test stimuli in this distribution will generally be different. In addition, the display situation of test stimulus *A* will include test stimulus *B*, and conversely. When direct matches are in question, the term *total display situation* will be used to cover the display situations of both test stimuli.

With indirect asymmetric matching, it is unnecessary to place any restriction on the absolute appearance criterion employed. However, not every proposition about the appearances of the two test stimuli, to be asserted or denied by the observer in a direct asymmetric comparison, leads to a matching relation between the test stimuli. The assertion must mean, in effect, that one test stimulus, *A* for example, in its display situation $(\text{ds} : x)$, possesses some appearance quality equal to that of the other test stimulus *B* in its display situation $(\text{ds} : y)$. If "dam : k" is used as an abbreviation for *matches in direct asymmetric comparison with criterion* k, the match is expressed symbolically by:

$$(A/\text{ds} : x) \text{ — dam : } k \text{ — } (B/\text{ds} : y)$$

where k must be an *equality criterion.* The equality criterion must be of such a kind that an interchange of the stimulus/display combinations leaves the truth of the above relation unaffected. In that case, the symmetrical relation is implied:

$$(B/\text{ds} : y) \text{ — dam : } k \text{ — } (A/\text{ds} : x)$$

A criterion expressing an inequality does not by itself define a matching relation. For example, a criterion with the assertion "$(A/\text{ds} : x)$ is brighter than $(B/\text{ds} : y)$" is no longer true on interchanging the stimulus/display combinations.

Also, not all equality criteria are acceptable. For example, affirming the proposition "$(A/\text{ds} : x)$ and $(B/\text{ds} : y)$ both have the hue yellow," corresponds certainly to an equality criterion, but it approaches closely the assertion of two independent propositions, "$(A/\text{ds} : x)$ is yellow" and "$(B/\text{ds} : y)$ is yellow," so that the appearances of the two test stimuli are not really being compared. The necessary condition can be put as follows: for direct asymmetric matching, the criterion must not only be an equality criterion; it also should be a strictly comparative judgment of the appearances of the two test stimuli, not separable into two judgments each having reference to the appearance of one test stimulus only. In this sense, the criterion of equality must be *nonseparable.*

Most investigations of matching properties employ direct asymmetric matching. The asymmetric procedures fall into two main groups. In the first group, the asymmetry enters incidentally when the two test stimuli are brought together in the same experiment. This facilitates much more precise comparisons of their visual appearance than can be obtained when absolute judgments are used in visual matching by strict substitution. The outstanding example is matching in a *bipartite* field. The two similarly placed, juxtaposed halves of the bipartite field are occupied respectively by simple test stimuli. The procedure is clearly asymmetric as the test stimuli are imaged on different, even though closely adjacent, retinal areas. Their positions with respect to other stimuli in the field are not precisely the same. Also the display situation for each test stimulus necessarily includes the other test stimulus. Thus, the two display situations differ in certain particulars.

Nevertheless, it is reasonable to assume in many cases that two test stimuli matched asymmetrically in this way would also match if substantially the same comparison could be made, with comparable precision, by a strictly symmetrical substitution procedure. Where this is so, the asymmetric procedure may be described as *quasi-symmetric.*

In the above example of the bipartite field, it is not difficult to make specific tests on a selection of the whole range of matches under study to determine whether the procedure qualifies as quasi-symmetric. In particular, one would try to ascertain:

(a) whether identical test stimuli applied respectively in the two half-fields yield a match,
(b) whether interchanging any two different but matched test stimuli occupying the respective half-fields, leaves the match intact,
(c) [given that (b) holds] whether simple transitivity is valid among sets of three matches: if A matches B and B matches C, then A matches C.

Affirmative results from these tests would indicate that the particular direct asymmetric matching procedure possessed the three characteristic properties of matching by strict substitution, namely, reflexivity, symmetry, and transitivity. The procedure would then qualify as quasi-symmetric. However, it must be noted that the set of all stimuli that are members of pairs in quasi-symmetric matching do not form an equivalence class. Transitivity in the wider sense, defined earlier, and valid for matching by strict substitution, does not hold. We expect quasi-symmetric matching if the following conditions are fulfilled:

(a) the areas of the retina responding to the two test stimuli have the same response properties,
(b) the other stimuli contributing to the display situation are so disposed as to affect equally the appearance quality being assessed for the two test stimuli,
(c) any possible effect of one test stimulus on the appearance of the other is at least the same for both when match is reached.

In the second main group of asymmetric comparisons, the emphasis is on the asymmetry. The objective is to determine how differences in the retinal area used in the conditioning stimuli and resulting adaptation conditions, and so forth, may affect the similarity of response to different test stimuli. The asymmetry is then an essential element in the comparison, and such matching procedures may be described as *specifically asymmetric.*

As previously explained, direct asymmetric matches will be obtained from the comparison if the observer's criterion is one of equality and is also nonseparable. But interchange of test stimuli, leaving the total display situation otherwise unchanged, will generally upset the match. The failure of this symmetry feature for direct specifically asymmetric matches prevents even the framing of the simple transitivity principle relating the matches, in pairs, of any three test stimuli, such as holds for quasi-symmetrical matching.

However, three modified forms of transitivity principle may hold in direct asymmetric matching. The first, *asymmetric transitivity for stimuli in direct asymmetric matches,* states that if any stimulus A_1 in display situation (ds : x) matches the two different stimuli B_1 and B_2 both in display situation (ds : y), and if B_1 in (ds : y) also matches A_2 in (ds : x), then B_2 does the same. In the adopted symbolic language, this statement reads as follows:

if $(A_1/\mathrm{ds} : x)$ — dam : k — $(B_1, B_2/\mathrm{ds} : y)$

and $(B_1/\mathrm{ds} : y)$ — dam : k — $(A_2/\mathrm{ds} : x)$

then $(B_2/\mathrm{ds} : y)$ — dam : k — $(A_2/\mathrm{ds} : x)$

If this holds, it follows that the corresponding relation with the roles of the two display situations interchanged will also be true. The other modified transitivity laws concern limited groups of different asymmetric matching procedures to be discussed in the next section.

5.2.3 Limited Groups of Asymmetric Matching Procedures

Particular examples of direct asymmetric matching procedures are briefly described in Section 5.2.5. However, first some consideration will be given to limited groups of such procedures, all using the same matching criterion but with different total display situations. It would be a useful concept in relating results obtained with the different procedures of such a group if it were permissible to think of the same (test stimulus/display situation)-combination participating in different procedures. It might then be found, for example, that there is simple transitivity in the matching of different combinations, just as there is for similar groups of indirect asymmetric matching procedures with a common matching criterion. The difficulty is that in a direct and specifically asymmetric match, the display situations of the two test stimuli are not independent. One display situation cannot be modified without, at least formally, modifying the other. However, suppose that in a particular matching procedure of the group, all the factors in the display situation of one test stimulus *(A,* for example) that were responsible for influencing *As* appearance, could be separated from the other factors. This would then enable one to define an *effective display situation,* abbreviated to (eds : x), but expressly excluding the other test stimulus as a possible influence. Suppose further that this could be done for both display situations in each matching procedure of the group. Again, one would assume that no factor in the effective display situation of one test stimulus would have an appreciable effect on the appearance of the other test stimulus and would not therefore belong to the effective display situation of the latter. Besides the test stimuli themselves, there will no doubt be other factors in the total display situation of any particular procedure that do not belong to either of the effective display situations of the two test stimuli. It is implied in the assumptions made that these factors are indifferent with regard to the appearances of the two test stimuli and have a negligible effect in modifying the matches made. These fairly demanding assumptions would justify the concept of independent combinations (test stimulus/effective display situation) or, in abbreviation, (ts/eds). They would allow for the same combination to occur in a number of different matching procedures.

However, the test stimuli themselves present certain problems. They have been excluded from the effective display situations on the tacit assumption that the effect on the appearance of any particular test stimulus resulting from the presence of any other test stimulus that it matches is likely to be slight. At least, the effect is not of such a kind as to vitiate the idea of the test stimulus in its effective display situation having a constant appearance in whatever match of the group it is concerned. This assumption is most plausible if the matching criterion calls for a complete color match.

An overall test of the validity of the foregoing concepts and assumptions for a particular group is provided by the *second modified transitivity principle for direct asymmetric matches.* This asserts transitivity among the (test stimulus/effective display)-combinations of a group:

if (A/eds : x) — dam : k — (B/eds : y)

and (B/eds : y) — dam : k — (C/eds : z)

then (C/eds : z) — dam : k — (A/eds : x)

A hypothetical example of a simple group of asymmetric matching procedures is illustrated in Figure 1(5.2.3), caption *(a).*

More practical and interesting groups of matching procedures are obtained if one does not insist that every pair of different (ts/eds)-combinations can be coupled together to form a matching procedure of the group. Instead, suppose that the (ts/eds)-combinations fall into two classes (I and II) and that the procedures of the group consist entirely of all the possible couplings of two combinations, one from each class. Then, the second modified transitivity principle that treats all (ts/eds)-combinations on an equal footing can no longer be formulated, but the *third modified transitivity principle for direct asymmetric matching* may be valid for the group. This states that

if

(A/eds : x) and (B/eds : y) belong to class I

(C/eds : z) and (D/eds : w) belong to class II

and (A/eds : x) — dam : k — (C/eds : z)

(B/eds : y) — dam : k — (C/eds : z)

and if (A/eds : x) — dam : k — (D/eds : w)

then (B/eds : y) — dam : k — (D/eds : w)

The above third modified transitivity principle for direct asymmetric matching may be referred to as the *asymmetric transitivity for the (test stimulus/effective display)-combinations of a two-class group of matching procedures.* Figure 1(5.2.3), Caption *(b),* illustrates the kind of two-class group that might meet the present conditions.

Embodied in all the transitivity principles applicable to direct matching, is the following idea: When each of two different test stimuli in the same display situation matches a particular com-

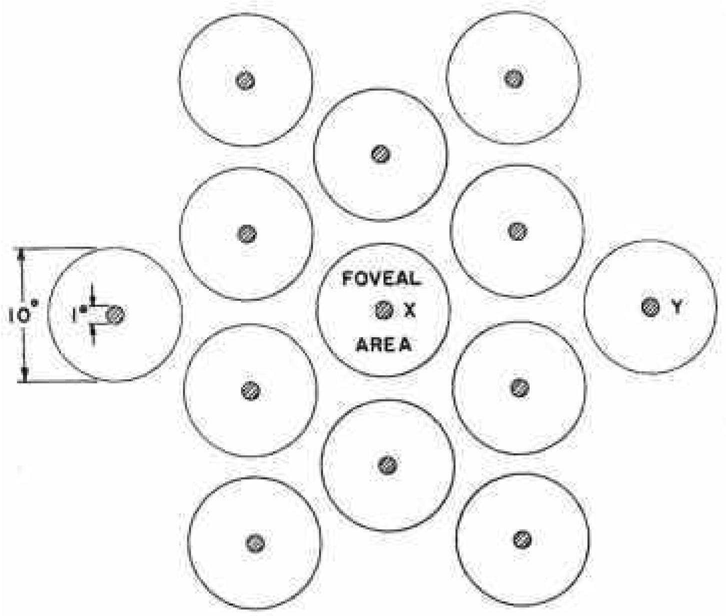


Fig. 1(5.2.3). **(*a*) Hypothetical single-class group of asymmetric matching procedures. The diagram shows 13 possible positions in the visual field for a 1° diameter test stimulus, each at the center of a uniform nonvariable illuminated surround of 10° diameter, the surrounds being all nonoverlapping. The (13·12)/1·2) — 78 asymmetric matching procedures of the group are obtained by using, in turn, every possible pair of test stimulus positions with their associated surrounds, the rest of the visual field being kept dark. Subject to the assumption that in each of the 78 matching procedures the appearance of either test stimulus is influenced solely by the absolute spectral radiant power distribution of its own surround stimulus, the group exemplifies the concept of a single class group with independent (ts/eds)-combinations. (*b)* Hypothetical two-class group of asymmetric matching procedures. By contrast with the group under (*a*), just one pair of test stimulus positions *(X* and F) with their associated surrounds is used for all matching procedures of the group. But in different procedures the absolute spectral radiant power distributions of the two surround stimuli are varied, independently, in any way desired. All the various surrounds about *X* and *Y*, respectively, that are included in the group correspond to the (ts/eds)-combinations of classes I and II.**

parison stimulus in its display situation, the appearance of the comparison stimulus is not different in the two matches because the test stimuli have different spectral compositions.

From what is known of the processes of vision through the initial absorption in visual pigments, we should certainly expect this to hold in most cases when complete color match is the matching criterion used. Where the criterion requires matching of a single appearance quality of the test stimuli, which may differ widely in other respects, it is more necessary to check whether any transitivity principle used in interpreting results does in fact hold.

Asymmetric matching is considered further in Sections 5.3, 5.4.12, and 5.12.1.

5.2.4 Matching Criteria

The criteria embodied in the visual judgments on which matching procedures are based vary widely in complexity. The simplest and most important is matching two light patches to *complete color match,* that is, in brightness, hue, and saturation. More complex are matching judgments that require the observer to mentally isolate a particular quality in the appearances of the two patches when these may differ in other respects. The principal judgments of this kind are the following:

(a) Matching brightness when the patches may differ in hue or saturation or both.

(b) Matching hue when brightness and saturation may differ.
(c) Matching saturation when brightness and hue may differ.
(d) Matching brightness and hue when the saturations of the patches may differ.
(e) Matching hue and saturation when the brightness may differ.
(f) Matching saturation and brightness when the hues may differ.

All these judgments of what may be called *incomplete color matches* have in fact been used to some extent, but judgment (a) is outstanding because it forms the basis of *heterochromatic photometry by the direct comparison method.* It is tacitly assumed in describing the above criteria that each of the light patches is homogeneous in appearance over its whole area. But this may not be the case even when the external stimuli applied to the respective test areas are uniform and steadily exposed. In some circumstances, notably with large (more than 2° diameter) foveally centered bipartite fields, inhomogeneities in patch appearance may necessitate the introduction of supplementary conditions in the matching criteria to enable the observer to make unambiguous judgments [see Examples (b) and (c) of Section 5.2.5]. An additional practical problem is that on first looking at the bipartite field, the color appearances may not be the same as after continued viewing, and a regular observational routine may have to be stipulated. With certain complex test stimuli, the light patches may have internal spatial or temporal structure that is perceived, but that is sufficiently fine for the observer to make mental averages of the appropriate qualities and apply the uniform-patch criteria. This complication is not unrelated to the everyday problem of matching the color of material surfaces of different textures.

The uniform-patch matching criteria have been commonly applied to symmetric bipartite fields. In the two halves of a bipartite field, simple test stimuli are imaged (in addition, possibly to a uniform conditioning stimulus) in total display situations such that a quasi-symmetric matching procedure is almost certainly assured. But they are also applicable in specifically asymmetric matching procedures in which the two test stimuli are in quite different display situations [see Examples (g), (h), and (i) in Section 5.2.5].

As well as matching the qualities of simple light patches treated as uniform, the eye can also assess the similarity of contrasts contained in the two patches to be matched between contiguous areas, particularly when the contrasts are weak. The criterion of equal contrast is applied to increase the precision of brightness matching in a bipartite field. By a suitable optical device, the radiant flux irradiating each half of the field also irradiates a patch (usually trapezoidal in shape) forming an enclave in the other half but to a stimulus level some 10% lower, although of the same relative spectral distribution. Figure 1(5.2.4) shows schematically the appearance of such a field introduced by 0. Lummer and E. Brodhun in the latter part of the last century (Walsh, 1958). The observer makes the match by adjusting the controls to equalize as closely as possible in all respects the contrasts presented by the trapezoids with their immediate surrounds.

The above photometric application of matching to equal contrasts employs, in a particular form, a more general matching criterion, the *matching of color differences,* contained respectively in two light patches on which are imaged complex stimuli. In the simplest case, each patch contains two similar uniform areas of different color, the two color differences being equalized in the match [see also Section 5.2.5(j)].

A matching judgment of a quite different kind is obtained when two geometrically similar light patches imaged on the same retinal area are presented in rapid alternation. By repeated observations and adjustments of the frequency of

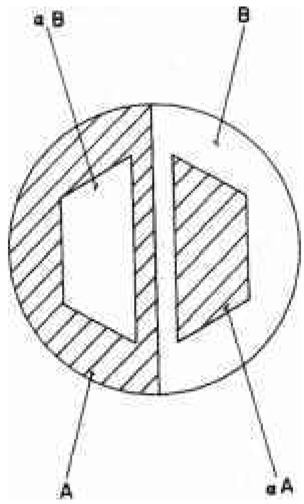


Fig. 1(5.2.4). Lummer-Brodhun photometric field of the contrast type. In the trapezoidal patches, the stimuli *A* and *B* are reduced by a factor α (typically, α = 0.9).

alternation and the radiance of one of the light patches, a condition is reached for which the matching field appears quiescent but begins to flicker if the radiance of the variable light patch is raised or lowered by a small amount. Although elaborate, the visual judgment or set of judgments involved here is in practice easy to apply and is effective whether or not the two light patches, when they are viewed in turn under steady conditions, have similar or widely different hues and saturations. The quality matched is commonly described as the "brightness" — perhaps better known as the *flicker brightness* — and provides the basis for the *flicker method of heterochromatic photometry* (Walsh, 1958).

Different in another way, but bearing some analogy with the minimal flicker criterion, is the use of the *distinctness of the straight-line border* between two contiguous simple test stimuli of different spectral compositions. This matching criterion stipulates that the distinctness of the border is minimal with respect to variations in either sense of a physical parameter controlling one of the test stimuli. If the relative spectral radiant power distributions of both test stimuli are kept constant, and the absolute radiant power (or luminance) of one of them is varied, the criterion provides a method of heterochromatic photometry yielding results similar to those obtained by the flicker method, but different from those obtained by the direct comparison method (Boynton and Kaiser, 1968). Distinctness of the border is used as matching criterion in a different way in the matching procedure employing two adjacent complex test stimuli, developed by Kaiser, Herzberg, and Boynton (1971), and Wagner and Boynton (1972). The two halves of each complex test stimulus are contiguous along a straight line and are occupied by unform stimuli of different relative or absolute spectral composition. The matching criterion is that the distinctness of the internal straight-line border is the same in the two test stimuli.

Any of the above criteria appropriate to the direct matching of two test stimuli can, of course, be used for indirect matching by the method of strict substitution. One of the two test stimuli is then kept fixed and forms part of the invariant display situation of the other, variable, test stimulus. However, in indirect matching by strict substitution, a very much wider range of matching criteria is available. In that case, the proposition defining the criterion does not involve the actual equating of some appearance quality of the test stimulus with that of a more or less similar stimulus in the invariant display situation. Two interesting visual judgments in this category were developed by Hurvich and Jameson (1955). One test stimulus only is entailed, and the observer judges whether its appearance (1) possesses neither the qualities of blueness nor yellowness, or (2) possesses neither the qualities of redness nor greenness. These criteria are absolute judgments that rest on the existence of the so-called *unique hues:* blue and yellow, red and green, and on the observer's ability to recognize for each of them whether the appearance of any given test stimulus partakes in some degree of the hue in question. Empirically, no stimulus appearance partakes of both blueness and yellowness, or of both redness and greenness. (The derivation of chromatic-response or chromatic-valence curves for spectral stimuli, using these criteria, is summarized in Section 5.4.12.)

Propositions defining criteria for indirect matching by strict substitution may correspond rather closely to those that lie at the root of the extensive development of *visual scaling.* For example, the observer may be required to assert that the test stimulus appears midway in color between a fixed red and a fixed white reference stimulus, all three being presented simultaneously in the visual field. Or, the observer may have to make the judgment that the test stimulus is, say, five times the brightness of a fixed reference stimulus. The numerical assessment of "sensation magnitudes," which seems to be involved in various judgments of this kind, presents certain logical difficulties, particularly to some observers. However, in practice, the degree of consistency of results obtained with visual judgments in these terms has enabled them to be applied successfully in applications and theoretical developments. (See Chapter 6 for material on scaling in color vision, not tied to the limited application to color matching procedures.)

One concluding remark may be made about matching criteria. Almost any statement about the appearance of a test stimulus could serve as a basis for a special kind of matching procedure. However, to be worthwhile, the procedure must yield results bearing on practical problems or be susceptible of interesting theoretical interpretation. Contrived matching procedures such as that mentioned earlier based on the visual judgment: "The test stimulus differs in hue from a given fixed comparison stimulus," can be of use only in illustrating extreme cases.

5.2.5 Some Particular Matching or Equivalence Procedures

The display situations and test stimuli for several matching or equivalence procedures employed in particular investigations are illustrated diagrammatically in Figure 1(5.2.5). The different procedures are labeled (a) to (m).

(a) Matching in a symmetric bipartite field small enough to be imaged in the rod-free foveal area of the retina. Wright (1929-1930) used monochromatic primary stimuli *R, G, B* to make complete color matches of monochromatic stimuli λ mixed with one of the primaries as desaturating stimulus *D*. His results, and closely similar measurements by Guild (1931) using primaries of wide-band spectral distributions, provided the spectral chromatid ty data on which the CIE 1931 Standard Colorimetric Observer is based [Sections 3.3.3(a) and5.15(ii)].

(b) Matching in a large, foveally centered symmetric bipartite field. With such large fields, the two halves of a central area of some 1° to 2° diameter may show a well defined color difference, both differing in color from the surrounding half-fields, when the latter are in good color match with each other. This occurs particularly when the stimuli being matched have widely different spectral compositions (Phenomenon of Maxwell Spot). Stiles (1955b) instructed observers to ignore the small central area and from their matches with spectral stimuli λ, suitably desaturated by *D*, derived complete color-matching functions. To reduce to probably insignificant amounts any intrusion of rod vision in the match, the primaries *R, G, B* were changed from red, green, and blue to red, yellow, and blue for the longer wavelength test stimuli, the data being finally transformed to a common reference set of primaries. A 14° diameter guard-ring surround *z* of the same spectral composition as the comparison stimulus *C*, separated from the matching field by a circular grey line, countered any disturbance of the match attributable to the high contrast edge between matching field and dark surround. This feature was probably of more importance in small field (2°) matching which was investigated concurrently [Sections 5.5.3, 5.5.4, and 5.15(ii)].

(c) Matching in a large foveally centered symmetric annular bipartite field. Speranskaya (1958, 1959) occluded a 2° diameter central area to eliminate match disturbance by the Maxwell Spot. She used a single set of primaries *R, G, B* throughout. Her data, after some correction for rod intrusion, were combined by Judd (CIE, 1960) with those of Stiles and Burch (1959) in the specification of the CIE 1964 Supplementary Standard Colorimetric Observer [Section 3.3.3(b)].

(d) Heterochromatic brightness matching by the minimal flicker method. Ives (1912) specified the field pattern (that shown) and other conditions best suited for this matching procedure. As the frequency of alternation of test *T* and comparison stimuli *C* of different chromaticities is increased, chromatic flicker disappears first. At a higher frequency, the optimal condition for application of the brightness criterion is reached. This frequency, selected by the observer, depends on the luminance level of the stimuli and on their respective chromaticities [Sections 5.7.1(i) and 5.7.2(i)].

(e) Matching by slow alternation of test and comparison stimuli in a single small test area. Alternation at a rate slow enough for the observer to assess separately the complete color appearances of test *T* and comparison stimuli *C* was used by Enoch and Stiles (1961) in determining the effect on the color of a monochromatic stimulus λ of changes in its angle of incidence on the retina. The main constituent of both test and comparison stimuli was monochromatic and of the same wavelength. Just sufficient mixtures of the primary stimuli *R, G, B* were added to each λ to enable satisfactory complete color matches to be made. The monochromatic component of each stimulus entered the eye pupil as a narrow pencil. The retinal angle of incidence obtained by displacing the point of entry in the pupil varied from normal to about 12° for the fully dilated pupil, a mydriatic being used, if necessary (Section 5.11).

(f) Matching in a symmetric bipartite field imaged on the extrafoveal retina. Clarke's investigations (1960a,b, 1963), in which the procedure illustrated was used (among others), was mainly directed to the breakdown of the additivity law for complete color matching in the extrafoveal retina and to the effects of rod participation in the match. The main complication is the tendency of the matching field to fade on steady fixation *FP* (Troxler's Effect), which is particularly marked when the field is imaged well outside the foveal area (Clarke, 1960a). The fading is largely avoided by exposing test *T* and comparison stimuli C simultaneously in brief observation periods that must, however, be long enough for color judgments to be made (Section 5.6.3).

(g) Matching of test and comparison stimuli imaged on small areas of widely different parts of the retina. In this procedure, the display situations of the stimuli being matched are clearly different because of the test locations on retinal areas with different properties. In addition, the separation θ of the areas makes it possible to introduce conditioning stimuli that can modify the observer's state of adaptation differentially. In Moreland and Cruz's work (1959), the pattern shown was used with azimuthal angles ψ of 0° and 90° and radial angles θ up to 50°. The comparison stimulus *C*, and *R, G, B* primary mixture, was imaged on the fovea. The quantities of these primaries in a match with a monochromatic test stimulus λ were used to calculate foveal chromaticities specifying the apparent color of the extrafoveal stimulus λ. Plotted in a diagram showing also the foveal spectrum locus for the same observer, these chromaticities display the progressive modification of color perception and its dimensionality with increasing extrafoveal angle θ (Section 5.6.3).

(h) Dichoptic matching with differential adaptation of the two retinal test areas in left and right eye, respectively. This method, pioneered by Wright (1934), was applied by him to compare the effects of chromatic adaptation on color matches during the recovery from different conditioning stimuli (*X* and *Y*) of the two eyes. In Hunt's procedure (1953), illustrated, a cyclic presentation (Period *A* followed by Period *B*) of conditioning and test stimuli was used. The results refer, in effect, to a constant adaptation at a particular time in the cycle. To facilitate the locking of the half-fields presented to the respective eyes in a dichoptic bipartite matching field, four fusion points *(FuP)* were constantly present in the fields of both eyes. Large and small (10° and 2° diameter) symmetric matching fields were used. The possibility of stimuli applied to one eye modifying the display situation of the test stimulus applied to the other eye arises. However, some experimenters, using comparable methods, believe such effects to be slight. Frequently, the dichoptic matching method is also referred to as *haploscopic matching* or as *binocular matching* [Section 5.12.2(i)].

(i) Matching with differential conditioning of the two halves of a foveally centered symmetric bipartite field. In the investigation by MacAdam (1956), accurate fixation on the center of the matching field was maintained throughout, and different conditioning stimuli (*X* and *Y*) were exposed in the respective halves of the matching field in intervals between the actual presentation and matching of the test *T* and comparison stimuli *C*. As *chromatic* adaptation was the main objective of the work, the luminances of the members of each of the six pairs of differently colored conditioning stimuli were approximately the same. Matching by MacAdam's method for each pair of conditioning stimuli may be regarded as a particular matching procedure. The use of numerous different pairs then represents a limited group of asymmetric matching procedures (Section 5.2.3). Whether the concept of *effective display situation* is applicable for this group and whether the *second modified transitivity principle for direct asymmetric matches* holds good is questionable. The answers turn on whether the conditioning of one half-field also affects the state of the other half-field so as to modify significantly the match. MacAdam's subsequent analysis of his data (1961, 1963) suggests by its form the validity of the effective display concept and the transitivity principle mentioned for the conditions used without perhaps establishing them (Stiles, 1967) [see also Section 5.12.2(v)].

(j) Matching the color difference between two contiguous test stimuli with the corresponding difference for a second copresent pair. In the work of Wyszecki (1965) and Wyszecki and Fielder (1971), the procedure, based on this matching concept, involved just three uniform hexagonal stimuli J_1, J_2, J_3 surrounded by a large white surround *z*. Each hexagon was filled by a variable mixture of the three instrumental primaries *R, G, B*. The observer has to assess three color differences in the stimulus array provided by the contiguous hexagon pairs (J_1, J_2), (J_1, J_3), (J_2, J_3). The final match is reached when all three hexagons appear equally bright and when their three color differences are judged to be equal. They then define in trichromatic color space a perceptually equilateral triangle on a surface of constant brightness (Section 7.10.6).

(k) Matching the distinctness of the straight-line borders contained in two adjacent complex stimuli. The simplest application of the distinctness of border concept as a method of heterochromatic

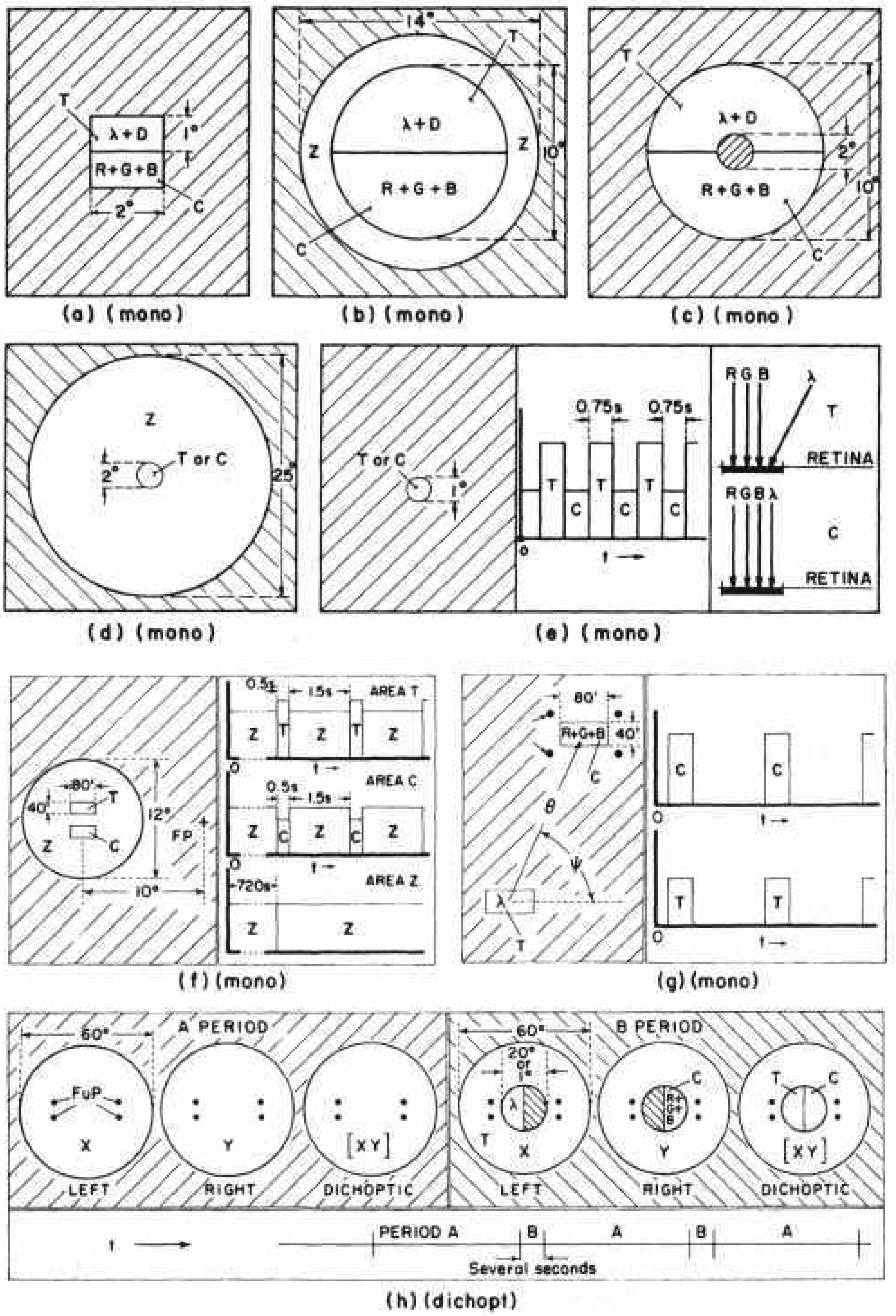
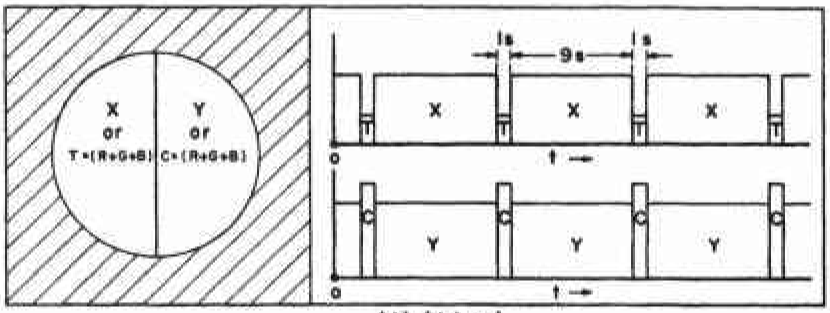
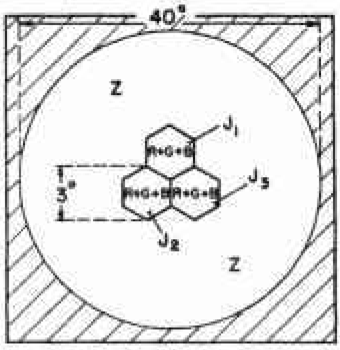


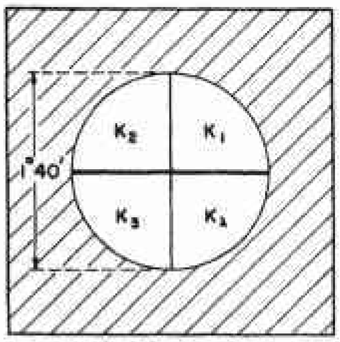
Fig. 1(5.2.5). Diagrams (*a*) to (*m*) illustrating the display situations and matching stimuli for various equivalence and matching procedures. Symbols used: *T:* test stimulus; *C*: comparison stimulus; *R, G, B:* set of primary stimuli; λ: monochromatic stimulus; *D:* desaturating stimulus; *I*: increment stimulus; J_1, J_2, J_3, K_1, K_2, K_3, K_λ: components in complex stimuli producing perceptual differences; *M:* variable stimulus in equivalence determinations; *X, Y, Z*: conditioning stimuli; *FP,OP,FuP*: small "light" points, and fusion points, all steadily exposed; hatched area: dark surround or dark field; *t:* time variable during observation; where no time sequence is shown, the observer is adapted to a steady stimulus pattern, apart from the test stimulus



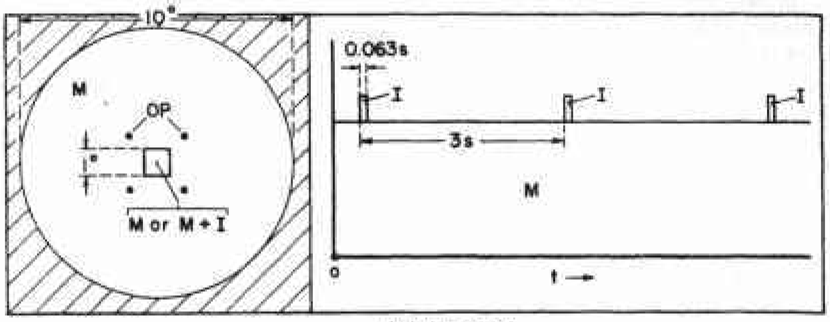
(i) (bino)



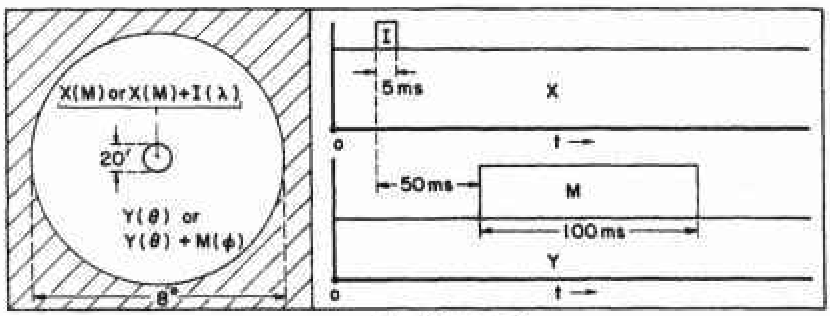
(j) (bino)



(k) (mono)



(l) (mono)



(m) (mono)

adjustments to reach a match. An exception is made in the case of flicker matching, where the rapid alternation in the test area is not illustrated; mono: the stimuli concerned in the match are exposed to one eye only, the other eye being kept in darkness or dim light (monocular viewing); bino: both eyes view the same stimulus pattern (binocular viewing); dichopt: the two eyes view somewhat different patterns, the match depending on what is seen in the combined binocular field (dichoptic viewing). Most of the diagrams are not to scale. In the time-sequence diagrams, the ordinate defines, in a limited sense, the magnitude of the indicated stimulus, showing when it is exposed and for how long.

matching involves only two contiguous simple test stimuli that are set to minimal distinctness of border (Boynton and Kaiser, 1968). The first step in a more searching study of border distinctness (Kaiser, Herzberg, and Boynton, 1971), illustrated in the diagram, was to vary the luminance of the monochromatic field K_λ to obtain minimal distinctness of the border with the fixed white field K_3 In further measurements, the contiguous stimuli K_3 and K_λ constituted one complex stimulus K_λ being set at various luminance levels above and below the value for minimal distinctness of the border with K_3 The second complex stimulus was provided by white-light stimuli K_2 and K_1 again presenting a straight-line border, the luminance of K_2 being made equal to that of K_3. Matching was carried out by varying the luminance of K_1 to equalize the distinctness of the vertical borders in the upper and lower semicircular fields. This could be done satisfactorily provided the resultant achromatic contrast $|K_2 - K_1| /(K_2 + K_1)$ was less than about 0.3 and above threshold. In this work, the observer must be equipped with an achromatizing lens to minimize the effects of chromatic aberrations of the eye. To reduce fading of the borders, the observer shifted fixation up and down between the middle and the upper and lower borders. In some experiments, the whole field was occluded for about 0.25 s in every 4 s. The results of the investigations led to a theory of the contribution of achromatic and chromatic differences to border distinctness.

(l) Determination of equivalent fields with respect to their effect on the detection of a fixed increment stimulus. In the procedure used by Stiles (1939), the relative spectral compositions of the variable field stimulus *M* and the fixed increment stimulus *I* are, in general, different (two-color threshold technique). The radiant power of the field stimulus *M* is determined at which *I* is at the threshold of detection. In practice, best results are obtained not by allowing the observer to raise and lower the field level to which the observer is supposed to be adapted, but by measuring increment thresholds at field levels giving values in the neighborhood of *I*, and by then interpolating to find the value corresponding to *I* precisely. The reciprocal powers of equivalent monochromatic fields define field sensitivities that, for appropriate choices of the fixed increment stimulus, may apply to different retinal response mechanisms (Sections 5.12.2 and 7.4).

(m) Determination of equivalent contrast pulses. As in the previous example, the criterion of equivalence is the effect of the detectability of a fixed increment stimulus *I* of a variable stimulus which, in this case, is an annular contrast pulse *M.* But the display situation of the increment pulse *I* is much more complex. Pulse *I* is presented for a brief period of time before the contrast pulse *M* which is intended to bring *I* to the threshold of detection. Also, additional conditioning stimuli *Y* and *X,* both steady, are applied respectively to the contrast pulse and increment stimulus areas. In this work, Alpern, Rushton, and Torii (1970) applied their technique to establishing the saturation, or upper limit, of the inhibitory effect of a contrast pulse on the detectability of an increment stimulus as the stimulus level in the annular field was raised by increasing 7. (The bracketed Greek letters in the diagram identify the various different-colored stimuli in the notation of Alpern et al.) The saturation effect was demonstrated for various sets of conditioning stimuli, designed respectively to confine the threshold response to the rod or one of several cone mechanisms. In doing this, the authors employed a modified form of equivalence criterion. The contrast pulse *M* on the steady field *X* had to raise the threshold of the increment stimulus to a fixed multiple of the value obtained when no contrast pulse was applied and when the only stimulus in the annular area was the steady field *X.* This procedure was devised to allow for the effect on the threshold of light scattered from *X* onto the central area (Section 7.6).

Six of the foregoing examples of matching procedures, namely (a) to (f), must be expected to satisfy the three conditions for quasi-symmetric matching. This will continue to hold good if the total display situation is changed by including additional or modified conditioning stimuli, either pre-exposed or steadily present, provided they are symmetrically arranged with respect to, and are likely to affect equally, the two halves of the bipartite field. Examples (g), (h), and (i) represent specifically asymmetric matching procedures. In Examples (j) and (k), the test stimuli are complex. The equivalence procedures (l) and (m) contrast a simple and a fairly elaborate display situation.

Among the many matching procedures unrepresented in Figure 1(5.2.5) the following may be mentioned:

(1) Dichoptic procedures in which the observer's viewing conditions are not so rigidly defined as in Example (h). A central septum may be used to separate two scenes containing simple test stimuli

in different display situations, viewed respectively by the left and the right eye of the observer. In making the match, the observer's eyes are free to wander, the left eye scanning one scene, the right eye the other (Winch and Young, 1951).

(2) Viewing conditions employing binocular vision and approximating very closely those of every day life. In work by Helson, Judd, and Warren (1952), observers were given preliminary training in which they learned to identify colored test samples, in a standard reference situation, by their Munsell specifications. Subsequently, they viewed colored test stimuli in different display situations and reported their apparent colors using the Munsell descriptions. This is a refinement of color-naming procedures in which nonspecially trained observers employ for description their ordinary equipment of color nomenclature. Procedures on these lines may be designated matching by long-term memory [see also Section 5.12.2(vii)].

(3) Newhall, Burnham, and Clark (1957) used a form of asymmetric matching designed to test how well colors are remembered over very brief periods. A color is presented alone for a limited period (5-10 s). Some five seconds later, a variable test color is set up by the subject (or selected from a range of samples) which is judged by memory to be in match with the first test color. The conditions for observation of the two test stimuli are closely similar, but the results show that the subject makes matches in which the saturation and luminance of the second test stimulus are appreciably greater than when the normal method of matching with simultaneous presentation is used. Newhall et al. were able to show that the asymmetric factor to which these differences are in the main attributable, is the matching of a stimulus actually being perceived with the short-term memory of a stimulus exposed a short time earlier.

(4) Complex test stimuli, produced by superimposing a resolvable dark bar grating over a uniform and minimally desaturated monochromatic test field, were used by Nunn (1977) to make complete color matches with a simple comparison test stimulus consisting of an adjustable *R, G, B* mixture. Even with identical display situations of the simple and complex test stimuli, large color differences were recorded, in some cases. Measurements were also made with different display situations that included internally structured or pre-exposed conditioning stimuli.

Any direct color-matching procedure (i.e., one in which test and comparison stimuli are presented to the observer in the same observation), reduces to matching by strict substitution if the comparison stimulus is kept constantly the same. The range of possible matches is thereby drastically limited, but this limitation may be useful in attempting to elucidate some of the causes for deviations from the ideal laws of trichromatic matching. The approach to color matching through such a simplifying restraint is dealt with in Section 5.3, Maxwell's method.

5.3 MAXWELL'S METHOD OF COLOR MATCHING

5.3.1 Historical Note

Maxwell in his original determination of his "mixture curves," which would now be called trichromatic color-matching functions, derived them from a set of matches in each of which a mixture of up to three different monochromatic stimuli was adjusted to match a fixed white-light stimulus (Maxwell, 1860a,b). By contrast, the principal later determinations of the trichromatic matching data for the average normal eye (Guild, 1931; Wright, 1928-1929; 1929-1930; Stiles, 1955b; 1958; 1963; Stiles and Burch, 1955; 1959; Speranskaya, 1958; 1959) have used mainly sets of color matches of the monochromatic stimuli, each desaturated with the minimal amount of one fixed primary stimulus necessary to make possible a match with a mixture of the remaining two fixed primary stimuli. If trichromatic color matching conformed completely to the linear laws of proportionality and additivity, deemed to hold for the CIE standard colorimetric observers, Maxwell's method and the second method, appropriately described by Crawford (1965) and Lozano and Palmer (1968) as the *maximum saturation method,* would give the same color-matching functions, although it is realized that in Maxwell's method the effect of random matching errors on the values of the derived functions will be magnified. However, if the validity of the linear laws in general trichromatic color matching is questionable, then the Maxwell method has the merit that all the matches used are made in a field of constant luminance and chromaticity and any nonlinearities caused by changes in the luminance and chromaticity of the matching field will be eliminated. Obviously, if linearity fails in some degree, no single set of color-matching functions will predict correctly all matches. The Maxwellian set might tend to give better predictions for